

Maths Calculation Guidance



Progression towards a standard method of calculation

December 2019

Introduction:

The 2014 National Curriculum provides a structured and systematic approach to the teaching of calculation. The aim is for mental calculations and written procedures to be performed efficiently, fluently, and accurately with understanding. Procedures and understanding are to be developed in tandem. End of key stage expectations are explicit in the programme of study.

At Allington Primary School, we have a consistent approach to the teaching of written calculation methods in order to ensure continuity and progression across the school.

Age related expectations:

This calculation policy is organised according to age appropriate expectations as set out in the National Curriculum 2014, however it is vital that pupils are taught according to the stage that they are currently working at, being moved onto the next level when ready, or working at a lower stage until they are secure enough to move on. Therefore headings link to steps, with the most relevant year group name in brackets.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods. It is also important for children to be confident to use mental and written strategies to explain their thinking. This must be a priority within calculation lessons. Written methods need to be viewed as tools to enable children to solve problems and record their thinking in an organised way.

Aims:

Children should be able to use an efficient method, mental or written appropriate to the given task, with understanding. By the end of year 6, children will have been taught, and be secure with, a compact standard method for each operation.

To develop efficient written calculation strategies children need:

- * Secure mental methods which are developed from early years
- * A solid understanding of the number system
- * Practical hands on experience including a range of manipulatives
- * Visual models and images including number lines and arrays
- * Experience of expanded methods to develop understanding and avoid rote learning
- * Secure understanding of each stage before moving onto the next.

Before carrying out a calculation, children will be encouraged to consider:

- * Can I do it in my head? (using rounding, adjustment)
- * The size of an approximate answer (estimation)
- * Could I use jottings to keep track of the calculation?
- * Do I need to use an expanded or compact written method?

Pre requisite skills for written calculations

Addition and subtraction:

- * Do they know all the addition and subtraction facts for all numbers to 20?
- * Do they understand place value and can they partition and then re-partition numbers?
- * Can they add three single digit numbers mentally?
- * Can they add and subtract any pair of two digit numbers mentally?
- * Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and Division:

- * Do they know the 2, 5 and 10 times tables and corresponding division facts?
- * Do they know the result of multiplying by 1 and 0?
- * Do they understand 0 as a place holder?
- * Can they multiply two and three digit numbers by 10 and 100?
- * Can they double and halve two digit numbers mentally?
- * Can they use multiplication and division facts they know to derive mentally other multiplication and division facts that they do not know?
- * Can they explain their mental strategies orally and record them using informal jottings?

These lists are not exhaustive but are a guide for the teacher as they structure the move from informal to formal methods of calculation. It is vitally important that children's mental methods of calculation are continued to be practised and secured alongside their learning and use of an efficient written method for each operation.

A pathway to teaching calculation methods:

Expanded methods should be viewed as steps towards a standard method and not as methods in themselves.

Before beginning to record in a more refined written format children must have had significant practical work reinforced with appropriate manipulative, models and images.

Teachers will guide pupils to refine their written methods of recording by modelling and asking questions such as “What is the same? What’s different?”

Learning will be planned to ensure pupils are encouraged to use and apply what they have learnt to problem solving tasks.

As children move along the pathway it is vital that they practice, reinforce, consolidate, use and apply it to mathematical learning and NOT simply move onto the next step.

Progression of learning in addition, subtraction, multiplication and division across KS1 and KS2

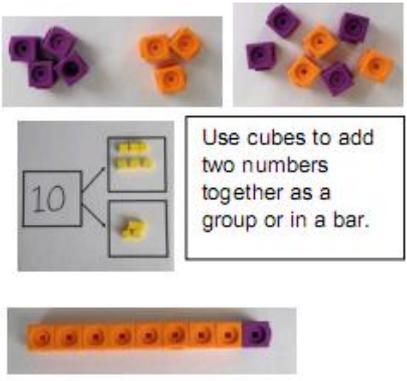
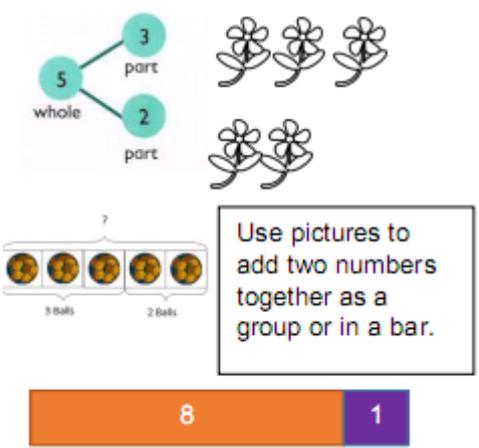
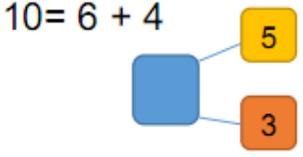
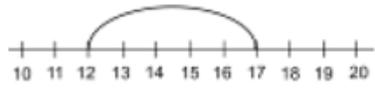
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

Step One (Year One) Addition and Subtraction

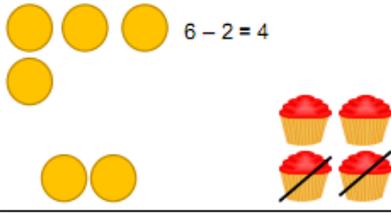
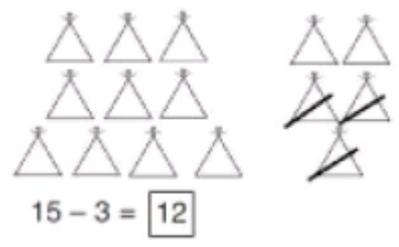
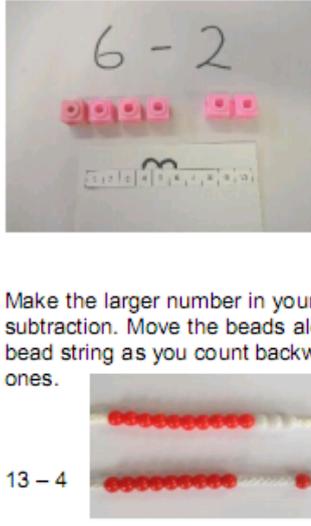
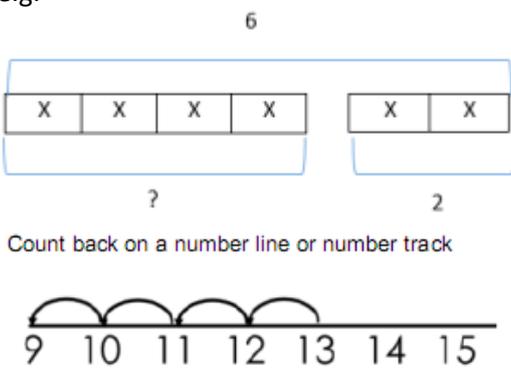
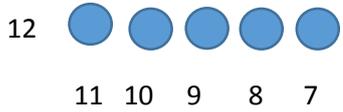
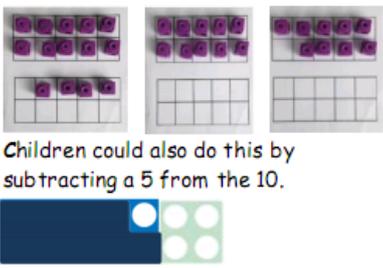
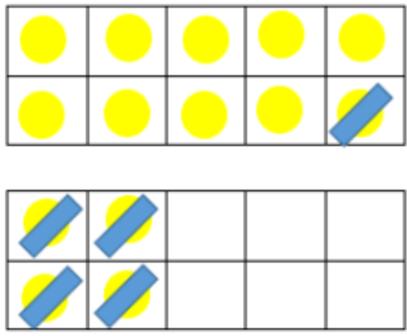
Addition and subtraction objectives:

- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = ? - 9$.

Addition Step One

Concrete	Pictorial	Abstract
<p>Use Numicon, denes, beads and Base 10 etc as a way of learning number bonds for pairs of numbers up to 10 and 20.</p>	<p>Record number bonds using + and = signs:</p>	<p>Combine two parts to make a whole:</p>
<p>Combine two parts to make a whole using multi-link cubes, Base 10, number beads and denes:</p>  <p>Use cubes to add two numbers together as a group or in a bar.</p>	<p>Combine two parts to make a whole:</p>  <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>Combine two parts to make a whole:</p> $4 + 3 = 7$ $10 = 6 + 4$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Starting at the larger number and counting on using number beads, Numicon, counters, multi-link cubes and Base 10:</p>  <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>Starting at the larger number and counting on:</p> $12 + 5 = 17$  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>Starting at the larger number and counting on, using jottings as markers</p> $12 + 5 =$ 

Subtraction Step One

Concrete	Pictorial	Abstract
<p>Physically taking away and removing objects from a whole</p>  <p style="text-align: center;">$6 - 2 = 4$</p>	<p>Cross out drawn objects to show what has been taken away</p>  <p style="text-align: center;">$15 - 3 = 12$</p>	<p>Written calculations using variation in layout:</p> <p style="text-align: center;">$18 - 3 = 15$</p> <p style="text-align: center;">$8 - 2 = 6$</p> <p style="text-align: center;">$\square = 8 - 2$</p>
<p>Counting back (using number lines or number tracks)</p>  <p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p> <p style="text-align: center;">$13 - 4$</p>	<p>Children to represent what they see pictorially e.g.</p>  <p style="text-align: center;">6 ? 2</p> <p style="text-align: center;">Count back on a number line or number track</p> <p style="text-align: center;">Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>	<p>Starting at the larger number and counting back, using jottings as markers</p> <p style="text-align: center;">$12 - 5 =$</p> <p>12</p>  <p style="text-align: center;">11 10 9 8 7</p>
<p>Making 10 (using numicon or ten frames)</p> <p style="text-align: center;">$14 - 5$</p>  <p>Children could also do this by subtracting a 5 from the 10.</p>	<p>Children to present the ten frame pictorially</p> 	<div style="border: 1px solid black; padding: 5px;"> <p>$14 - 5 = 9$ You also want children to see related facts e.g. $10 - 9 = 5$</p> <p>Children to represent how they have solved it e.g.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>$14 - 5 = 9$</p>  </div> <div style="text-align: center;"> <p>14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5</p> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p>$14 - 5 = 9$</p>  </div> <div style="text-align: center;"> <p>5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9</p> </div> </div> </div>

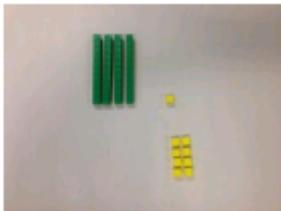
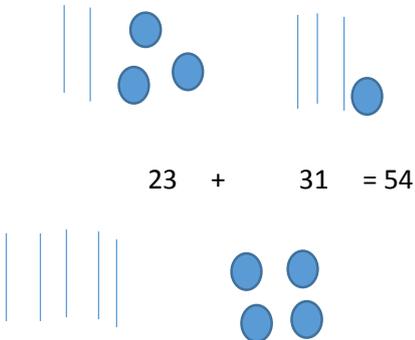
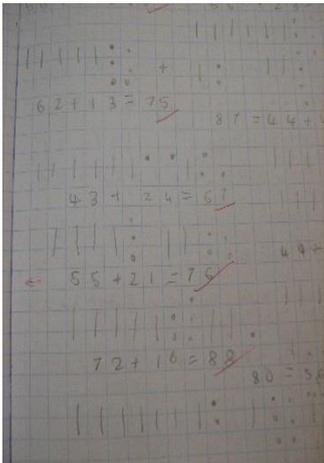
Step Two (Year Two) Addition and Subtraction

Addition and subtraction objectives:

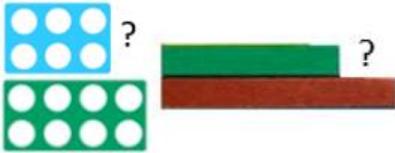
- solve problems with addition and subtraction:
 - 1) using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - 2) applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Vocabulary: +, add, addition, more, plus, make, sum, total, altogether, score, double, near double, one more, two more... ten more... **one hundred more**, how many more to make...? How many more is... than...? How much more is...? – subtract, subtraction, take (away), minus leave, how many are left/left over? One less, two less... ten less... **one hundred less**, how many fewer is... than...? How much less is...? Difference between, half, halve, = equals, sign, is the same as **tens boundary, inverse**

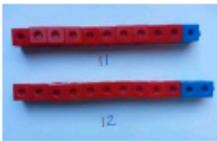
Addition Step Two

Concrete	Pictorial	Abstract
<p>T + O using base 10. Continue to develop understanding of partitioning and place value</p> <p>$41 + 8$</p> 	<p>Children to represent the concrete using a particular symbol e.g. lines for tens and circles for ones (chips and peas)</p>  <p style="text-align: center;">$23 + 31 = 54$</p>	<p>Symbolic recording using jottings</p> 

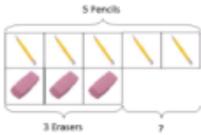
Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used)



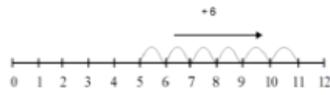
Compare amounts and objects to find the difference.



Use cubes to build towers or make bars to find the difference



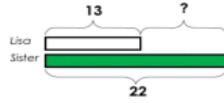
Use basic bar models with items to find the difference



Count on to find the difference.

Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.

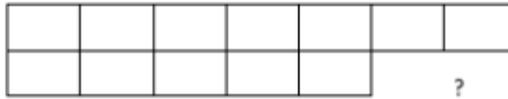


Draw bars to find the difference between 2 numbers.

Children to draw the cubes/other concrete objects which they have used

XXXXXXXXX
XXXXXX

Use of the bar model



Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Find the difference between 8 and 6.

8 - 6, the difference is ?

Children to also explore why $9 - 7 = 8 - 6$ (the difference, of each digit, has changed by 1 do the difference is the same)

Step Three (Year Three) Addition and Subtraction

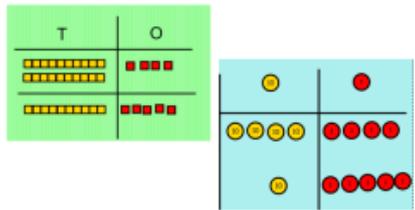
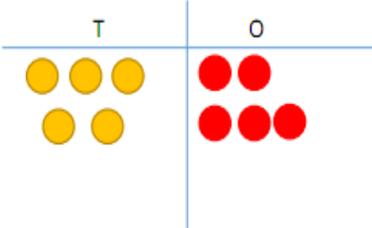
Addition and subtraction objectives:

- add and subtract numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Vocabulary: +, add, addition, more, plus, make, sum, total, altogether, score, double, near double, one more, two more... ten more... one hundred more, how many more to make...? How many more is... than...? How much more is...? – subtract, subtraction, take (away), minus, leave, how many are left/left over? One less, two less... ten less... one hundred less, how many fewer is... than...? How much less is...? Difference between, half, halve = equals, sign, is the same as tens boundary, **hundreds boundary**

Addition Step Three

Continuing on using ‘chips and peas’ as set out in the Year 2 guidance to provide transition from Year 2, moving onto other representations of base 10, using place value counters.

Concrete	Pictorial	Abstract
<p>Column method – no regrouping using place value counters and base 10:</p> <p>$24 + 15 =$ Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p> 	<p>Column method – no regrouping:</p> <p>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p> 	<p>Column method – no regrouping:</p> <div style="border: 1px dashed black; background-color: #e0f2f1; padding: 10px; margin: 10px 0;"> <p style="text-align: center;"><u>Calculations</u></p> $21 + 42 =$ $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$ </div>
<p>Column method – regrouping using place value counters and Base 10:</p>	<p>Column method – regrouping:</p>	<p>Column method – regrouping:</p>

Make both numbers on a place value grid.

146
+ 527

Add up the units and exchange 10 ones for one 10.

146
+ 527

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.

Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

$$\begin{array}{r} 267 + 85 \\ \underline{+85} \\ 352 \\ 11 \end{array}$$

Subtraction Step Three

Concrete

Column method – without regrouping

Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.

Pictorial

Draw the Base 10 or place value counters alongside the written calculation to help to show working.

Abstract

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

This will lead to a clear written column subtraction.

Column method – with regrouping

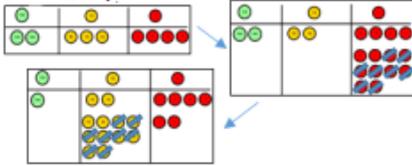
Column method (using base 10 and having to exchange)

45-26



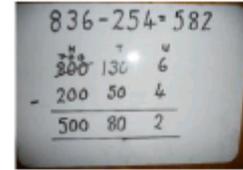
- 1) Start by partitioning 45
- 2) Exchange one ten for ten more ones
- 3) Subtract the ones, then the tens.

Column method (using place value counters) 234-88

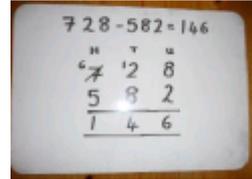


Hundreds	Tens	Ones
5	12	6
- 2	7	5
3	5	1

Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.

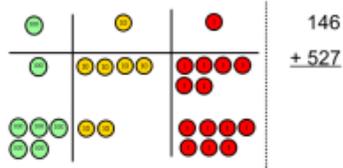
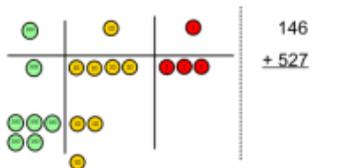
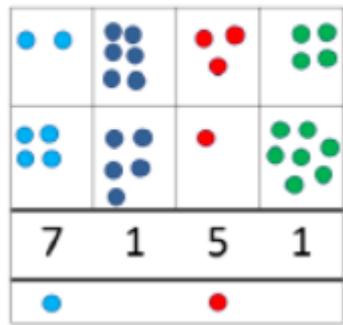
Step Four (Year Four) Addition and Subtraction

Addition and subtraction objectives:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Vocabulary: add, addition, more, plus, **increase**, sum, total, altogether, score double, near double, how many more to make...? Subtract, subtraction, take (away), minus, **decrease**, leave, how many are left/left over? **Difference between**, half, halve, how many more/fewer is... than...? How much more/less is...? Equals, sign, is the same as, tens boundary, hundreds boundary **inverse**

Addition Step Four

Concrete	Pictorial	Abstract
<p>Column method – regrouping:</p> <p>Make both numbers on a place value grid.</p>  <p>Add up the units and exchange 10 ones for one 10.</p>  <p>Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.</p> <p>This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.</p> <p>As children move on to decimals, money and decimal place value counters can be used to support learning.</p>	<p>Column method – regrouping:</p> <p>Children can draw a pictorial representation of the columns and place value counters to further support the learning and understanding.</p> 	<p>Column method – regrouping:</p> <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 0 auto;"> $\begin{array}{r} 7836 \\ +1351 \\ \hline 9187 \\ 1 \end{array}$ </div>

Subtraction Step Four

Concrete

Column method – with regrouping

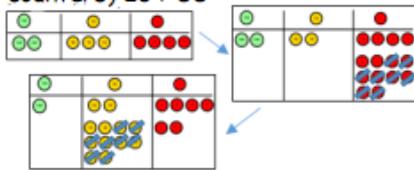
Column method (using base 10 and having to exchange)

45-26

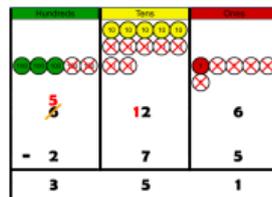


- 1) Start by partitioning 45
- 2) Exchange one ten for ten more ones
- 3) Subtract the ones, then the tens.

Column method (using place value counters) 234-88

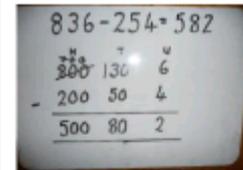


Pictorial

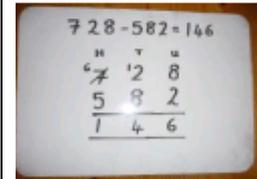


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

Abstract



Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.

$$\begin{array}{r}
 \overset{2}{2}\overset{1}{3}4 \\
 - \quad 88 \\
 \hline
 \quad \quad 6 \\
 \hline
 \end{array}$$

Step Five (Year Five) Addition and Subtraction

Addition and subtraction objectives:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Vocabulary: add, addition, more, plus, increase, sum, total, altogether, score double, near double, how many more to make...? Subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? Difference between, half, halve, how many more/fewer is... than...? How much more/less is...? Equals, sign, is the same as, tens boundary, hundreds boundary, **thousands, millions boundary, units boundary, tenths boundary**, inverse

Addition Step Five

Concrete and Pictorial

Use physical/pictorial representations alongside columnar methods where needed.

Mental methods like partitioning and recombining:

$$23\ 457 + 3\ 430$$

$$23\ 457 + 3000 + 400 + 30$$

$$26\ 457 + 400 + 30$$

$$26\ 857 + 30$$

$$26\ 887$$

Place value counters to support column addition and subtraction

$234\ \text{kg} + 49\ \text{kg} = 273\ \text{kg}$ $\begin{array}{r} 200 + 30 + 4 \\ \quad 40 + 9 \\ \hline 200 + 70 + 13 \end{array}$

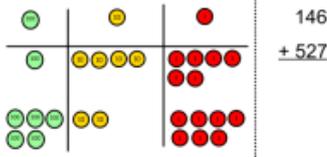
Abstract

$\begin{array}{r} 35871 \\ +16751 \\ \hline 52622 \\ \hline 111 \end{array}$
--

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

When adding and subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

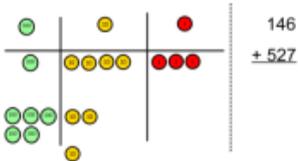
Make both numbers on a place value grid.



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

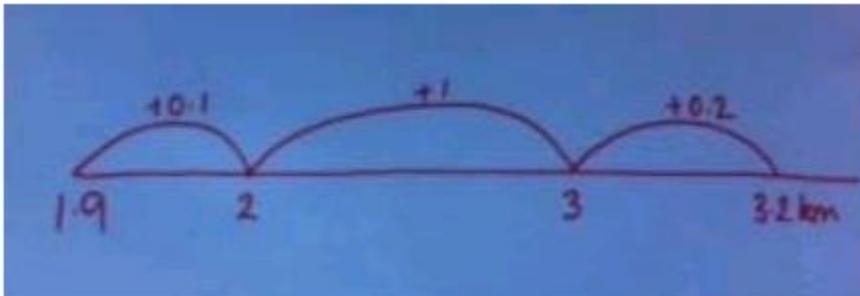
This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

Add up the units and exchange 10 ones for one 10.



As children move on to decimals, money and decimal place value counters can be used to support learning.

Empty number lines (particularly with time questions):



$$\begin{array}{r} 401.20 \\ + 26.85 \\ \hline 428.05 \\ \hline 428.76 \end{array}$$

They will also be:

- adding several numbers with different numbers of digits, understanding the place value;

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \hline 1121 \end{array}$$

- adding and subtracting decimals (with mixed numbers of decimal places), knowing that the decimal points line up under one another.
- Adding and subtracting amounts of money and measures, including those where they have to initially convert from one unit to another.

Subtraction Step Five

Concrete and Pictorial

Abstract

Use physical/pictorial representations alongside columnar methods where needed.

Children should continue to use the decomposition method to solve calculations such as:

Place value counters

Number lines

Partitioning

$$\begin{array}{r} \cancel{7}^6 \\ - 3 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 6 \\ 7 \\ - 3 \\ \hline 3 \end{array}$$

$$\begin{array}{r} \cancel{3}^2 \\ - 1 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 6 \\ 7 \\ - 3 \\ \hline 3 \end{array}$$

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals (with numbers having different numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Step Six (Year Six) Addition and Subtraction

Year Six addition and subtraction objectives:

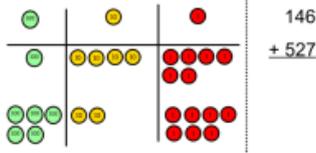
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Vocabulary: add, addition, more, plus, increase, sum, total, altogether, score double, near double, how many more to make...? Subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? Difference between, half, halve, how many more/fewer is... than...? How much more/less is...? Equals sign =, is the same as, tens boundary, hundreds boundary units boundary, tenths boundary **hundredth boundary**, **thousandths boundary**, inverse

Addition Step Six

Concrete	Pictorial	Abstract
<p>Use physical/pictorial representations alongside columnar methods where needed.</p> <p>Mental methods such as partitioning and recombining:</p> <p>23 457 + 3 430</p> <p>23 457 + 3000 + 400 + 30</p> <p>26 457 + 400 + 30</p> <p>26 857 + 30</p> <p>26 887</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> $234 \text{ kg} + 49 \text{ kg} = 273 \text{ kg}$ $\begin{array}{r} 200 + 30 + 4 \\ \quad 40 + 9 \\ \hline 200 + 70 + 13 \end{array}$ </div> <p>Place value counters to support column addition and subtraction</p>		<div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> $\begin{array}{r} 23587 \\ +21675 \\ \hline 45262 \\ \quad 111 \end{array}$ </div> <p>Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.</p> <p>When adding and subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.</p>

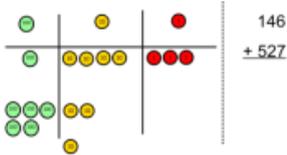
Make both numbers on a place value grid.



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

Add up the units and exchange 10 ones for one 10.



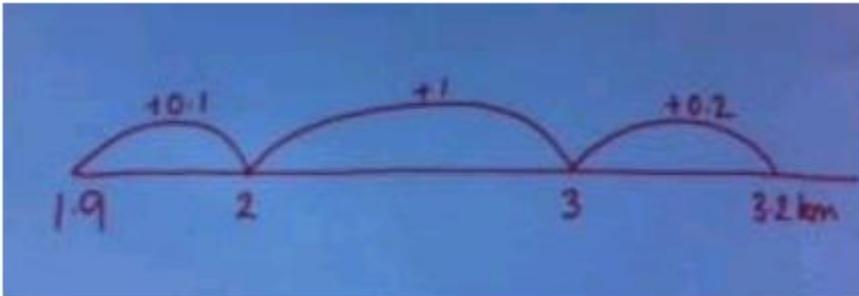
As children move on to decimals, money and decimal place value counters can be used to support learning.

$$\begin{array}{r}
 401.20 \\
 + 26.85 \\
 \hline
 428.76 \\
 \hline
 \end{array}$$

They will also be:

- adding several numbers with different numbers of digits, understanding the place value;

Empty number lines (particularly when solving time problems):



$$\begin{array}{r}
 42 \\
 6432 \\
 786 \\
 3 \\
 + 4681 \\
 \hline
 11944 \\
 \hline
 1121
 \end{array}$$

- adding and subtracting decimals (with mixed numbers of decimal places), knowing that the decimal points line up under one another. Adding and subtracting amounts of money and measures, including those where they have to initially convert from one unit to another.

Subtraction (Step Six)

Concrete and pictorial

Abstract

Use physical/pictorial representations alongside columnar methods where needed.

Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

Place value counters

$$\begin{array}{r}
 5 \quad 13 \\
 \cancel{6} \cancel{4} \quad 13 \quad 2 \\
 - 4681 \\
 \hline
 1751
 \end{array}$$

Number lines

Partitioning

$$\begin{array}{r}
 \overset{3}{4} \overset{1}{1} \overset{6}{7} . \overset{11}{2} \overset{1}{0} \\
 - \quad \quad 3 \quad 4 . 7 \quad 1 \\
 \hline
 3 \quad 8 \quad 2 . 4 \quad 9
 \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

Step One (Year One) Division and Multiplication

Multiplication and Division objectives:

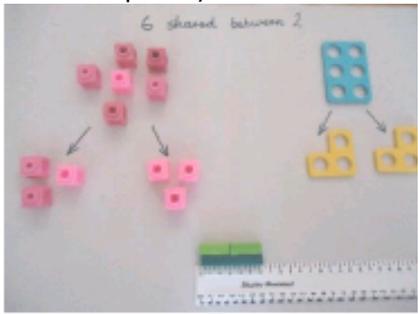
- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Vocabulary: lots of, groups of \times , times, multiply, multiplied by multiple of once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens, equal groups of, \div , divide, divided by, divided into left, left over

Multiplication Step One

Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4</p> 	<p>Children to represent the practical resources in a picture e.g.</p> <p>XX XX XX XX XX XX</p> <p>Use of a bar model for a more structured method</p> 	<p>3×4</p> <p>$4 + 4 + 4$</p>

Division Step One

Concrete	Pictorial	Abstract		
<p>6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates)</p> 	<p>Two circles, each containing three small circles, representing 6 shared between 2.</p> <p>This can also be done in a bar so all 4 operations have a similar structure:</p> 	<p>$6 \div 2 = 3$</p> <p>What's the calculation?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="width: 50px; text-align: center;">3</td> <td style="width: 50px; text-align: center;">3</td> </tr> </table>	3	3
3	3			

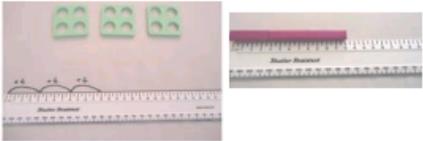
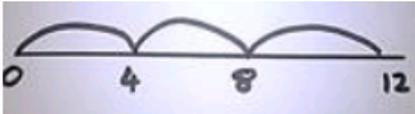
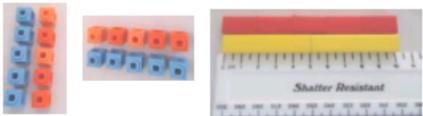
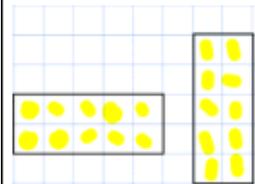
Step Two (Year Two) Multiplication and Division

Multiplication and Division objectives:

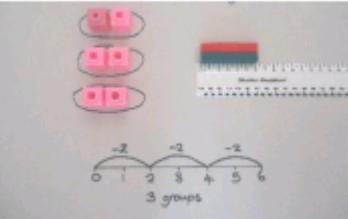
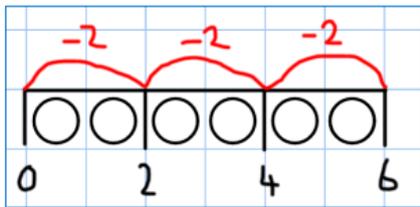
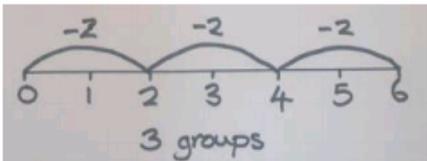
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Vocabulary: lots of, groups of \times , times, multiply, multiplied by multiple of once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of \div , divide, divided by, divided into left, left over

Multiplication Step Two

Concrete	Pictorial	Abstract
<p>Use number lines to show repeated groups e.g 3×4</p> 	<p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line $3 \times 4 = 12$</p> 
<p>Use arrays to illustrate commutatively (counters and other objects can also be used) $2 \times 5 = 5 \times 2$</p> 	<p>Children to draw the arrays</p> 	<p>Children to be able to use an array to write a range of calculations e.g.</p> <p>$2 \times 5 = 10$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $5 + 5 = 10$</p>

Division Step Two

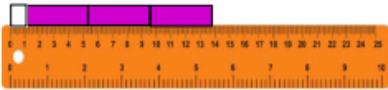
Concrete	Pictorial	Abstract
<p>Understand division as repeated grouping and subtracting $6 \div 2$</p> 		<p>Abstract number line</p> 

2 digit number divided by a 1 digit number with a remainder:

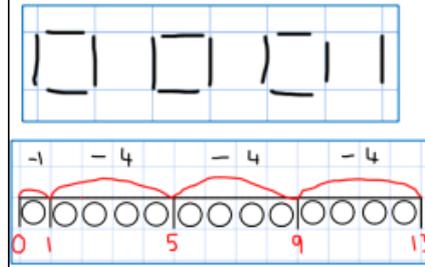
Use of lollipop sticks to form wholes



Use of Cuisenaire rods and rulers (using repeated subtraction)

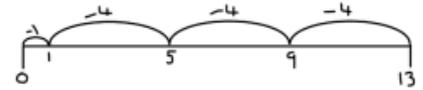


Children to have chance to represent the resources they use in a pictorial way e.g. see below:



$$13 \div 4 = 3 \text{ remainder } 1$$

Children to count their times tables facts in their heads



Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation $13 \div 4$, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each.

How many can I buy?

Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4.

There are 13 apples. How many boxes are needed?

Answer: 4 (the remaining 1 apple still needs to be placed into a box)

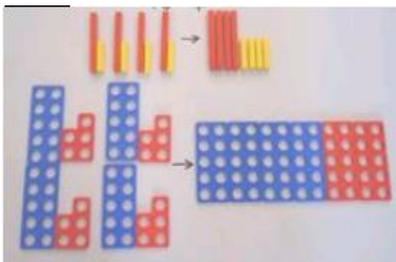
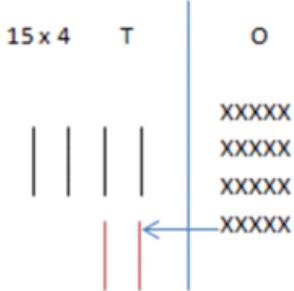
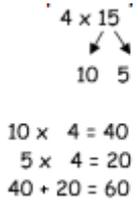
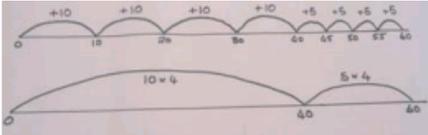
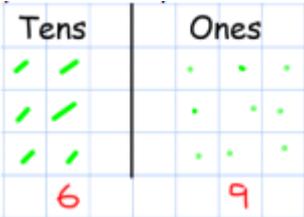
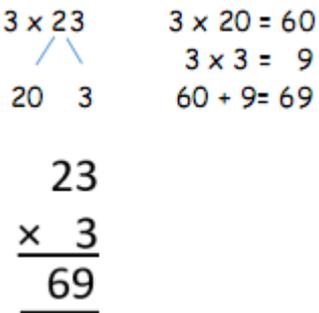
Step Three (Year Three) Multiplication and Division

Multiplication and Division objectives:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Vocabulary: lots of, groups of \times , times, multiply, **multiplication**, multiplied by multiple of, **product**, once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition, array, row, column double, halve, share, share equally, one each, two each, three each... group in pairs, threes... tens, equal groups of, \div , divide, **division**, divided by, divided into, left, left over, **remainder**

Multiplication Step Three

Concrete	Pictorial	Abstract
<p>Partition to multiply (use Numicon, base 10, Cuisenaire rods) 4×15</p> 	<p>Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like:</p> 	<p>Children to be encouraged to show the steps they have taken</p>  <p>4×15 $10 \quad 5$ $10 \times 4 = 40$ $5 \times 4 = 20$ $40 + 20 = 60$</p> <p>A number line can also be used</p> 
<p>Formal column method with place value counters or base 10 (at the first stage-no exchanging) 3×23</p> <p>Make 23, 3 times. See how many ones, then how many tens</p> 	<p>Children to represent the counters in a pictorial way</p> 	<p>Children to record what it is they are doing to show understanding</p>  <p>3×23 $3 \times 20 = 60$ $20 \quad 3$ $3 \times 3 = 9$ $60 + 9 = 69$</p> <p style="text-align: center;">23 $\times 3$ $\hline 69$</p>

Division Step Three

Concrete	Pictorial	Abstract
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Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$43 \div 8 =$

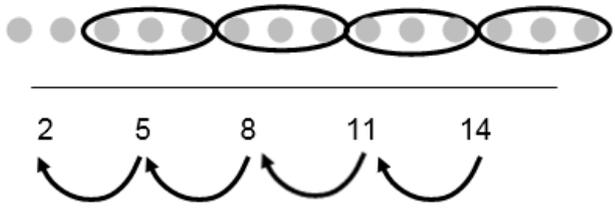


$43 \div 8 = 5 \text{ remainder } 3$

Using a number line the children will count on in an amount equal to the divisor. They will then count how many 'jumps' they made.

Example

$14 \div 3 = 4 \text{ remainder } 2$



Children will use the language of 'remainder'.

Grouping objects and drawing number lines gives a good visual understanding of remainders.

Children will become familiar with interpreting the remainders, when faced with 'real life' problems.

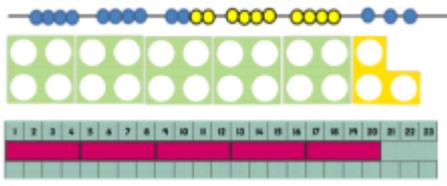
Instead of writing 'remainder', children may also abbreviate this and write 'r'.

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

A range of resources can be used e.g. bead counters, Numicon, Cusinaire rods

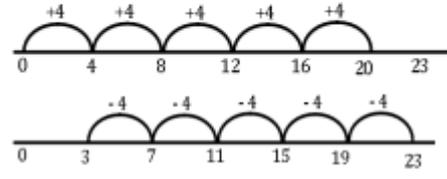
Using resources.

$23 \div 4 = 5 \text{ r}3$



Repeated addition and subtraction along a number line.

$23 \div 4 = 5 \text{ r}3$



Begin to solve mentally.

$23 \div 4 = \square$
 $31 \div 6 = \square$

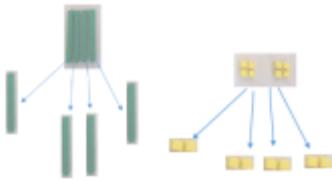
Missing number problems.
 $\square \div 3 = 4 \text{ r}1$
 $17 \div \square = 3 \text{ r}2$

Two digit numbers divided by 1 digit (no remainders) using base 10. Partition the two digit number into tens and ones and then divide by the single digit.

Children to represent the base 10 and sharing pictorially.

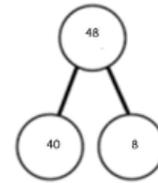
Partition the numbers into tens and ones and then divide by the single digit:

$$48 \div 4 = 12$$



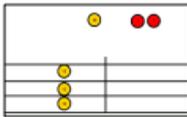
Start with the tens.

$$48 \div 4$$

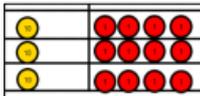


4 tens \div 4 = 1 ten
 8 ones \div 4 = 2 ones
 $10 + 2 = 12$

Sharing using place value counters.
 $42 \div 3 = 14$

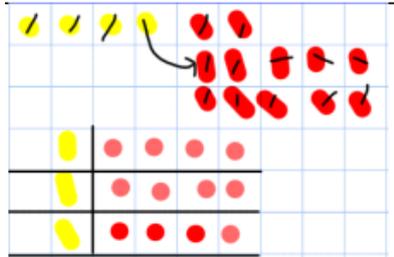


1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?



Exchange the ten for 10 ones and share out 12 ones

Pictorial representations e.g.



Moving onto abstract calculations with partitioning using known number facts e.g.

$$42 \div 3$$

$$42 = 30 + 12$$

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

$$10 + 4 = 14$$

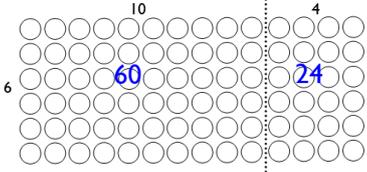
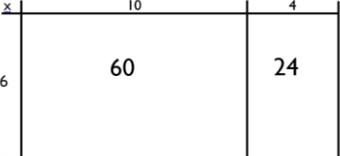
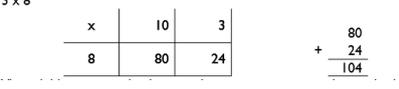
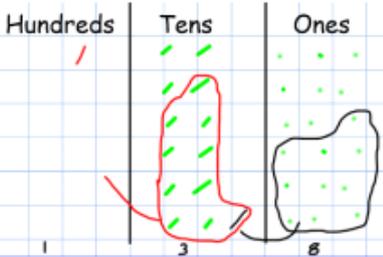
Step Four (Year Four) Multiplication and Division

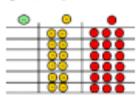
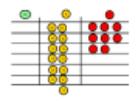
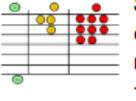
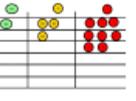
Multiplication and Division objectives:

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Vocabulary: lots of, groups of times, multiply, multiplication, multiplied by multiple of, **facts to 12×12** , product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of divide, division, divided by, **divided into remainder factor, quotient, divisible by, inverse**

Multiplication Step Four

Concrete	Pictorial	Abstract
<p>Using practical resources e.g. counters, children can use previous knowledge of arrays to make the grid method:</p> 	<p>By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.</p>  <p>This can progress onto the grid method:</p> 	<p>The calculation can then be recorded as the abstract written form as shown below.</p>
<p>Formal column method with place value counters (children need this stage, initially, to understand how the column method works)</p>	<p>Children to represent the counters/base 10, pictorially e.g. the image below.</p> 	<p>The aim is to get to the formal method but the children need to understand how it works.</p>

<p>6×23</p>  <p>Step 1: get 6 lots of 23</p>  <p>Step 2: 6×3 is 18. Can I make an exchange? Yes! Ten ones for one ten....</p>  <p>Step 3: 6×2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred...</p>  <p>Step 4- what do I have I each column?</p>		$6 \times 23 =$ $\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ \hline 11 \end{array}$
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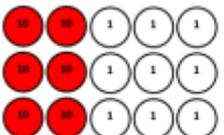
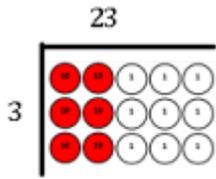
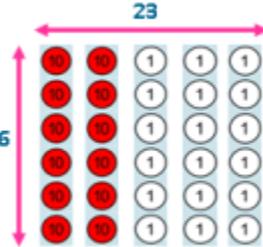
Children on step four will then progress onto using a formal standard written method for multiplication:

- $TU \times U$ (units first) * Leading to $HTU \times U$

$$\begin{array}{r} 23 \\ \times 7 \\ \hline 161 \\ 2 \end{array}$$

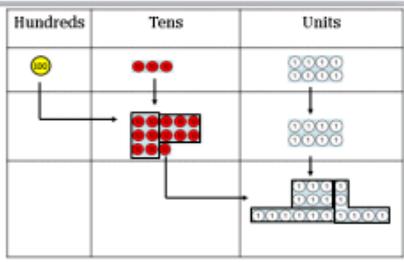
$$\begin{array}{r} 452 \\ \times 6 \\ \hline 2712 \\ 31 \end{array}$$

Division Step Four

Concrete	Pictorial	Abstract
<p>Use place value counters and set out in short division form e.g.</p> <p>$69 \div 3 = 23$</p>  <p>Check using multiplication inverse: $23 \times 3 = 69$</p>	<p>Extending divisions to resemble written method of short division.</p>  <p>$69 \div 3 = 23$</p>	<p>Short division.</p> $\begin{array}{r} 23 \\ 3 \overline{)69} \end{array}$
<p>$TU \div U$ and $HTU \div U$ where exchanging is required:</p>	<p>Result of grouping/sharing counters during 'concrete' stage.</p>  <p>Check using multiplication inverse.</p>	<p>Short division methods.</p> $\begin{array}{r} 023 \\ 6 \overline{)138} \\ \underline{6} \\ 13 \\ \underline{12} \\ 18 \\ \underline{18} \\ 0 \end{array}$ <p>$138 \div 6 = 23$</p> <p>432 ÷ 5 becomes</p> $\begin{array}{r} 86r2 \\ 5 \overline{)432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$ <p>Answer: 86 remainder 2</p>

Grouping and sharing using place value counters.
 Exchanging counters which cannot be grouped.

$$138 \div 6 = 23$$



Step Five (Year Five) Multiplication and Division

Multiplication and Division objectives:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non- prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Vocabulary: lots of, groups of, times, multiply, multiplication, multiplied by, multiple of, product, once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition, array, row, column, double, halve, share, share equally, one each, two each, three each... group in pairs, threes... tens, equal groups of, divide, division, divided by, divided into remainder factor, quotient, divisible by, inverse,

Multiplication Step Five

Concrete

Pictorial

Abstract

Children on step five will continue to use concrete and pictorial methods where necessary but the focus will be moving them onto a standard written method for multiplication:

First by multiplying a four digit number by a one digit using the short written method e.g.

$$\begin{array}{r} 1625 \\ \times 6 \\ \hline 9750 \\ 313 \end{array}$$

Then multiplying four digit numbers by a two digit number using the long written method e.g.

$$\begin{array}{r} 352 \\ \times 27 \\ \hline 2464 \\ 7040 \\ \hline 9504 \\ 1 \end{array}$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

$$4.62 \times 3 = ?$$

This method will then lead to multiplication of numbers involving decimals. Estimation by rounding will be encouraged first.

$$\begin{array}{r} 4.62 \\ \times 3 \\ \hline 13.86 \\ 1 \end{array}$$

Division Step Five

Concrete

Pictorial

Abstract

Children on step five will continue to use concrete and pictorial methods where necessary but the focus will be moving them onto a standard written method for division:

Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds.

615 ÷ 5

Step 1: make 615

Step 2: Circle your groups of 5

Step 3: Exchange 1H for 10T and circle groups of 5

Step 4: exchange 1T for 10ones and circles groups of 5

The use of concrete and pictorial methods can also be used for dividing decimals.

The use of the short written method:

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

Some children will require pictorial and concrete support but the main aim will be for children to use a standard written method for short division.

Dividing numbers up to four digits by a one digit number:

Showing remainder as a whole number:

$$\begin{array}{r} 858 \text{ r } 2 \\ 3 \overline{) 2576} \end{array}$$

Answer: 858 remainder 2

Showing remainder as a fraction:

$$\begin{array}{r} 858 \text{ r } 2 \\ 3 \overline{) 2576} \end{array}$$

Answer: $858 \frac{2}{3}$

Problem solving will involve rounding in context: There are 432 guests at a wedding. Each table at dinner seats 15 people. How many tables are needed?

Step Six (Year Six) Multiplication and Division

Multiplication and Division objectives:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations

Vocabulary: lots of, groups of times, multiply, multiplication, multiplied by multiple of, product, once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition, array, row, column, double, halve, share, share equally, one each, two each, three each... group in pairs, threes... tens, equal groups of, divide, division, divided by, divided into remainder, factor, quotient, divisible by, inverse

Multiplication Step Six

Concrete	Pictorial	Abstract
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Children on step 6 should be confident using the abstract written methods for multiplication, however for those who need additional support concrete and pictorial methods should be used alongside the abstract method (see earlier steps.) Demonstration of modelling each calculation can also be shown alongside the written method using brackets as seen in this TU x TU example:

$$46 \times 19 = ?$$

$$\begin{array}{r} 46 \\ \times 19 \\ \hline \end{array}$$

Step 1: Multiply top amount x units

Step 2: Multiply top amount x tens

Step 3: Add the answers together

$$\begin{array}{r} 414 \\ 460 \\ \hline 874 \end{array}$$

(46x9)
(46x10)

These are for demonstration only to remind children exactly what is happening.

Children on step 6 will be able to use both the short and written methods to **multiply 4 digit numbers by 1 and 2 digit numbers.**

They should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

Division Step Six

Concrete

Pictorial

Abstract

Children will divide numbers up to 4 digits by a one and two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Showing remainder as a whole number:

$$\begin{array}{r} 858 \text{ r } 2 \\ 3 \overline{) 2576} \end{array}$$

Answer: 858 remainder 2

Showing remainder as a fraction:

$$\begin{array}{r} 858 \text{ r } 2 \\ 3 \overline{) 2576} \end{array}$$

Answer: $858 \frac{2}{3}$

Children will then progress into changing the remainder into its decimal form by adding a decimal point and a zero in the tenths column.

Problem solving will involve rounding in context: There are 432 guests at a wedding. Each table at dinner seats 15 people. How many tables are needed?

Division of a two digit number:

Children will first use the short division method to divide a number by a two digit number and then progress onto a long division method. The ready reckoner method will be used alongside to support children e.g. (1x, 2x, 3x, 5x, 10x)

Concrete and pictorial guidance (for those children who need additional support):

Use place value counters and pictures to represent the long division method for dividing a number by two digits:



$$2544 \div 12$$

How many groups of 12 thousands do we have? None

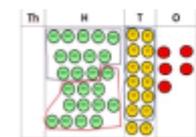


Exchange 2 thousand for 20 hundreds.

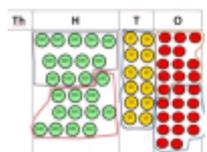


How many groups of 12 are in 25 hundreds? 2 groups. Circle them.

We have grouped 24 hundreds so can take them off and we are left with one.



Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.



Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

$$12 \overline{) 2544}$$

Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.

$$12 \overline{) 2544}$$

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

$$12 \overline{) 2544}$$

Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.

$$12 \overline{) 2544}$$

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

Moving onto decimals:

432 ÷ 15 becomes

$$15 \overline{) 432.0}$$

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Answer: 28.8